

Laser vector measurement technique for the determination and compensation of volumetric positioning errors. Part I: Basic theory

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A new laser vector method for the measurement of the volumetric positioning errors of a computer numerically controlled machine or a precision instrument is described here. Compared with conventional laser interferometer measurement, the laser vector method measures the vector errors, namely, the displacement error, vertical straightness error, and horizontal straightness error, rather than the displacement error only. The key to the laser vector method is that the measurement direction or the laser beam direction is not parallel to the displacement direction. With four setups, all three displacement, six straightness, and three squareness errors can be determined. Once the volumetric positioning errors are measured, they can be used to compensate for the repeatable positioning errors of the machine or instrument. The basic concept, theory, and measurement errors are described. Experimental verification of the vector method is in the Part II of this article. © 2000 American Institute of Physics. [S0034-6748(00)04010-7]

I. INTRODUCTION

The linear displacement errors, straightness errors, squareness errors, angular errors, and nonrigid body errors determine the performance or accuracy of a computer numerically controlled (CNC) machine tool, a coordinate measuring machine (CMM), or a precision instrument. Characterization of a machine movement is very complex. For each axis of motion, there are six errors, three linear errors, and pitch, yaw, and roll angular errors in the X, Y, and Z directions. For a three-axis machine, there are 18 errors plus 3 for squareness, a total of 21 errors. Complete measurement of these errors is very time consuming. Body diagonal measurements have been recommended¹ for a quick check of the volumetric accuracy. This is because it is sensitive to all the error components. However, if the errors measured are large, there is not enough information to identify the error sources.

Machine accuracy can be improved by measuring all these errors and then compensating for these errors, provided that the machine is repeatable.²⁻⁴ The key is how to measure these errors accurately and quickly. There are many methods by which to measure these errors. Zhang *et al.*⁵ measured the displacement errors along 22 lines in the machine work zone to determine the volumetric errors. Beckwith⁶ used a complex five laser beam system, in which two of the laser beams were used for displacement interferometers and three of the laser beams were used for lateral displacements with quadrant photodetectors. However, all of these methods are very complex and time consuming.

Described here is a laser vector measurement technique. It can measure all these errors using a simple and portable laser interferometer or a laser Doppler displacement meter (LDDMTM),⁷ in four setups and within a few hours.

11. BASIC CONCEPT

To measure the displacement accuracy of a linear axis, a laser interferometer can be used. The laser beam is aligned

parallel to the motion of the linear axis and the position errors are measured at each increment. Since the measurement direction is parallel to the direction of movement, the measured displacement errors are not sensitive to the straightness errors which are perpendicular to the displacement direction.

It is noted that for a quick check of volumetric positioning accuracy linear displacement measurement along four body diagonals is recommended by the B5.57 standard¹ This is because the body diagonal measurements are sensitive to all the errors such as the displacement errors, straightness errors, squareness errors, and angular errors. Hence it is a good check of volumetric accuracy. However, if the measured errors are large, there are not enough data to identify the sources of the errors.

The basic concept of the laser vector measurement technique is that the laser beam direction (or the measurement direction) is not parallel to the motion of the linear axis. Hence, the measured displacement errors are sensitive to the errors both parallel and perpendicular to the direction of the linear axis. More precisely, the measured linear errors are the vector sum of errors, namely, the displacement errors (parallel to the linear axis), the vertical straightness errors (perpendicular to the linear axis), and horizontal straightness errors (perpendicular to the linear axis and the vertical straightness error direction), projected to the direction of the laser beam. Furthermore, by collecting data with the laser beam pointing in three different diagonal directions, all nine error components can be determined. Since the errors of each axis of motion are the vector sum of the three perpendicular error components, we call this measurement a "vector" measurement technique.

In practice, first point the laser beam in one of the body diagonal directions, the same as in the body diagonal measurement. Instead of moving x, y, and z continuously to the next increment R, stop and take a measurement. Now, move

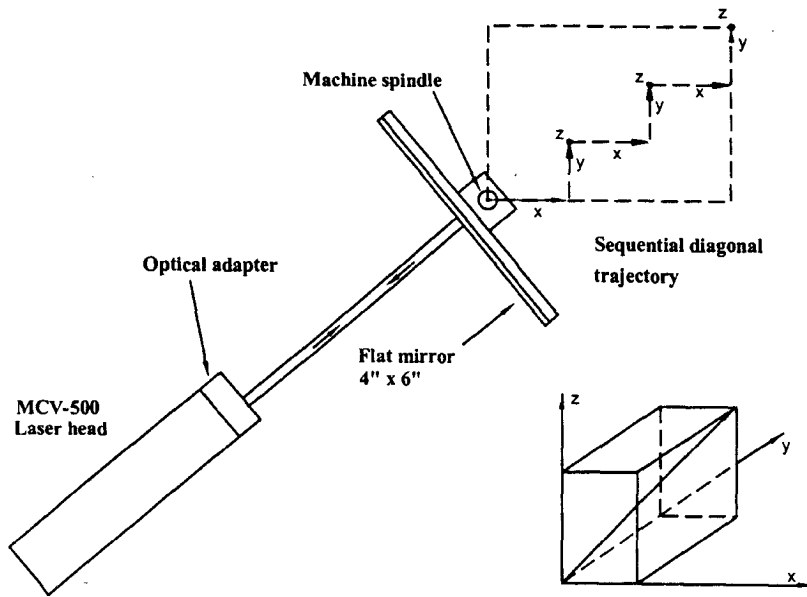


FIG. 1. Schematic of the vector measurement setup.

the x axis to X , stop and take a measurement, then move the y axis to Y , stop and take a measurement, then move the z axis to Z , stop and take a measurement. Here $R = \sqrt{X^2 + Y^2 + Z^2}$ is the increment in the diagonal direction, and X , Y , and Z are the increments in the x , y , and z directions, respectively.

Compared to the conventional body diagonal measurement where only one data point is collected at each increment R , the vector measurement collects three data points, one at X , one at Y , and one at Z . Hence three times more data are collected. Furthermore, the data collected after X are due to x -axis movement only, and data collected after Y and Z are due to y -axis and z -axis movement, respectively. Hence the error sources due to x -axis motion, y -axis motion, and z -axis motion can be separated.

Second, point the laser beam into another body diagonal direction and repeat the same process until all four body diagonals are measured. Since each body diagonal measurement collected 3 sets of data, there are 12 sets of data. Hence, there are enough data to solve the three displacement errors and six straightness errors.

For conventional body diagonal measurement, the displacement is a straight line along the body diagonal; hence a laser interferometer can be used to do the measurement. However, for the vector measurement described here, the displacements are along the x axis, then along the y axis, and then along the z axis. The trajectory of the target or the retroreflector is not parallel to the diagonal direction. Deviations from the body diagonal are proportional to the size of the increment, X , Y , or Z . A conventional laser interferometer will be way out of alignment even with an increment of a few mm.

To tolerate such a large lateral deviation, a laser Doppler displacement meter⁷ using a single aperture laser head and a flat mirror as the target can be used. This is because any lateral movement or movement perpendicular to the normal direction of the flat mirror will not displace the laser beam. Hence the alignment is maintained. After three movements, the flat-mirror target will move back to the center of the

diagonal again, hence the size of the flat mirror need only be larger than the largest increment. A schematic showing the vector measurement setup is shown in Fig. 1. Here the flatmirror target is mounted on the machine spindle and it is perpendicular to the laser beam direction.

Compared to a conventional body diagonal measurement all three axes move simultaneously along a body diagonal and collect data at each preset increment. In the vector measurement all three axes move in sequence along a body diagonal and collect data after each axis is moved. Hence, not only three times more data are collected, the error due to the movement of each axis can also be separated.

III. BASIC THEORY

A. Motion of a rigid body

The general motion of a rigid body along one axis can be described by six degrees of freedom. These are one linear, two straightness, one pitch, one yaw, and one roll. For a three axis machine, there are 18 degrees of freedom plus three squareness, a total of 21 degrees of freedom.

For each axis of motion, there are linear position errors, straightness errors, and pitch, yaw, and roll angles in the x , y , and z directions. Hence, for x -axis movement, there are linear position error $\delta_x(x)$, straightness errors $\delta_y(x)$ and $\delta_z(x)$, and pitch, yaw, and roll angles $\alpha_y(x)$, $\alpha_z(x)$, and $\alpha_x(x)$.

Similarly for y -axis and z -axis movement, there are linear position errors $\delta_y(y)$ and $\delta_z(z)$, straightness errors $\delta_x(y)$, $\delta_x(z)$, $\delta_y(z)$, and pitch, yaw, and roll angles $\alpha_x(y)$, $\alpha_z(y)$, $\alpha_y(y)$, $\alpha_x(z)$, $\alpha_y(z)$, and $\alpha_z(z)$.

The squareness between axes is θ_{xy} , θ_{yz} , and θ_{zx} .

B. Assumptions

To simplify the analysis, the following assumptions are made.

- (a) The motion is repeatable to within certain uncertainty. The accuracy of the method is limited to the repeatability of the motion.

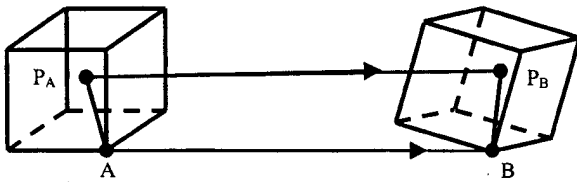


FIG. 2. Rigid body motion from point A to point B.

- (b) The position errors can be superpositioned, i.e., the position error is much smaller than the travel distance.
- (c) The angular errors are small compared to the other errors.

For most machine tools or CMMs, the above assumptions are a good approximation.

C. Trajectory model

The general motion of a rigid body starting from point A and ending at point B can be described by six degrees of freedom. These are one linear position error, two straightness errors, and three angular errors as shown in Fig. 2.

To simplify the analysis, pick representative point Pa on the rigid body (such as the tool tip or probe tip), and move the coordinate such that, at A, Pa is at the origin of the coordinate. Assume the motion is along the x direction with an increment of X and move the origin of the new coordinate to X. If there is no error, Pb should be at the origin of the new coordinate. However, in general Pb is not at the origin. As shown in Fig. 3, in the new coordinate at B, Pb = Xu_x + E(x), (a boldface letter indicates a vector quantity) where u_x is the unit vector in the x direction and E(x) is the vector position error (or volumetric error) due to motion in the x direction. In general E(x) can be expressed as

$$\mathbf{E}(x) = E_x(x)\mathbf{u}_x + E_y(x)\mathbf{u}_y + E_z(x)\mathbf{u}_z, \tag{1}$$

Where u_x, u_y, and u_z are unit vectors in the directions of the x, y, and z axes, E_x(x) is the error component in the x direction due to the motion in x, and E_y(x), and E_z(x) are the error components in the y and z directions, respectively, due to the motion in x. Please note that E_x(x), E_y(x), and E_z(x) are the position error components due to all the motion errors including the position error, two straightness errors, three angular errors, and even nonrigid body motion errors. Similarly, the errors due to y-axis motion and z-axis motion are E(y) and E(z), respectively, and can be expressed as

$$\begin{aligned} \mathbf{E}(y) &= E_x(y)\mathbf{u}_x + E_y(y)\mathbf{u}_y + E_z(y)\mathbf{u}_z, \\ \mathbf{E}(z) &= E_x(z)\mathbf{u}_x + E_y(z)\mathbf{u}_y + E_z(z)\mathbf{u}_z. \end{aligned} \tag{2}$$

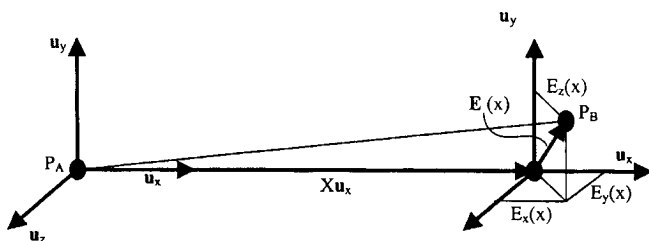


FIG. 3. Volumetric displacement errors or a vector error.

D. Measurement along the body diagonal

Assume the measurement is along a diagonal direction R with increments of X, Y, and Z. The unit vector R can be expressed as

$$\mathbf{R} = X/R\mathbf{u}_x + Y/R\mathbf{u}_y + Z/R\mathbf{u}_z. \tag{3}$$

The displacement error dR measured along the diagonal direction is the position error vector E projected toward the diagonal direction R. Hence it is the scalar product of E and R. That is,

$$dR = \mathbf{E} \cdot \mathbf{R} = E_x \frac{x}{R} + E_y \frac{y}{R} + E_z \frac{z}{R}, \tag{4}$$

where · means a scalar product of two vectors.

More specifically,

$$\begin{aligned} dR(x) &= E_x(x) \frac{x}{R} + E_y(x) \frac{y}{R} + E_z(x) \frac{z}{R}, \\ dR(y) &= E_x(y) \frac{x}{R} + E_y(y) \frac{y}{R} + E_z(y) \frac{z}{R}, \\ dR(z) &= E_x(z) \frac{x}{R} + E_y(z) \frac{y}{R} + E_z(z) \frac{z}{R}, \end{aligned} \tag{5}$$

where dR(x) is the displacement error measured along the diagonal direction due to movement of the x axis, and dR(y) and dR(z) are the displacement errors measured along the diagonal direction due to movement of the y and z axes, respectively.

E. Measurement along the four body diagonals

There are four diagonals, namely, those from (0, 0, 0) to (nX, nY, nZ), denoted by ppp, those from (nX, 0, 0) to (0, nY, nZ), denoted by npp, those from (0, nY, 0) to (nX, 0, nZ), denoted by pnp, and those from (0, 0, nZ) to (nX, nY, 0), denoted by ppn, where n is the number of increments, ppp means all increments are positive, npp means all increments are positive except X, pnp means all increments are positive except Y, and ppn means all increments are positive except Z.

Let dR(x)ppp be the displacement error measured along the ppp diagonal direction due to movement of the x axis. The first equation of Eq. (5) becomes

$$\begin{aligned} dR(x)_{ppp} &= E_x(x) \frac{x}{R} + E_y(x) \frac{y}{R} + E_z(x) \frac{z}{R}, \\ \text{Similarly, for the other diagonals,} \\ dR(x)_{npp} &= -E_x(x) \frac{x}{R} + E_y(x) \frac{y}{R} + E_z(x) \frac{z}{R}, \\ dR(x)_{pnp} &= E_x(x) \frac{x}{R} - E_y(x) \frac{y}{R} + E_z(x) \frac{z}{R}, \\ dR(x)_{ppn} &= E_x(x) \frac{x}{R} + E_y(x) \frac{y}{R} - E_z(x) \frac{z}{R}. \end{aligned} \tag{6}$$

Solving Eq. (6) for E_x(x), E_y(x), and E_z(x), we have

$$\begin{aligned}
 E_x(x) &= [dR(x)_{ppp} - dR(x)_{npp}] \frac{R}{2X}, \\
 E_y(x) &= [dR(x)_{ppp} - dR(x)_{pnp}] \frac{R}{2Y}, \\
 E_z(x) &= [dR(x)_{ppp} - dR(x)_{ppn}] \frac{R}{2Z}.
 \end{aligned} \tag{7}$$

Similarly for y -axis and z -axis movement,

$$\begin{aligned}
 E_x(y) &= [dR(y)_{ppp} - dR(y)_{npp}] \frac{R}{2X}, \\
 E_y(y) &= [dR(y)_{ppp} - dR(y)_{pnp}] \frac{R}{2Y}, \\
 E_z(y) &= [dR(y)_{ppp} - dR(y)_{ppn}] \frac{R}{2Z}, \\
 E_x(z) &= [dR(z)_{ppp} - dR(z)_{npp}] \frac{R}{2X}, \\
 E_y(z) &= [dR(z)_{ppp} - dR(z)_{pnp}] \frac{R}{2Y}, \\
 E_z(z) &= [dR(z)_{ppp} - dR(z)_{ppn}] \frac{R}{2Z}.
 \end{aligned} \tag{8}$$

By substituting the measured displacement along all four diagonals, dR_{ppp} , dR_{npp} , dR_{pnp} and dR_{ppn} , into Eqs. (7) and (8), the position errors, $E_x(x)$, $E_y(x)$, $E_z(x)$, $E_x(y)$, $E_y(y)$, $E_z(y)$, $E_x(z)$, $E_y(z)$, and $E_z(z)$ can be calculated.

F. Relation between the measured volumetric errors and the conventional 21 errors

As shown by Schultshik⁴ and by Zhang et al.,⁵ the volumetric error is the difference between the actual spindle position and the true spindle position. The true spindle position can be calculated by the coordinate transformations of each axis movement. Since we place the coordinate system on the tip of the tool or probe, the tool offset is zero. For a machine type $FXYZ$, the position errors can be expressed as

$$\begin{aligned}
 P_x &= \delta_x(x) + z\alpha_y(x) - y[\alpha_z(x) + \theta_{xy}] + \delta_x(y) + z[\alpha_y(y) \\
 &\quad - \theta_{xz}] + \delta_x(z), \\
 P_y &= \delta_y(x) - z[\alpha_x(x) + \theta_{yz}] + \delta_y(y) - z\alpha_x(y) + \delta_y(z), \\
 P_z &= \delta_z(x) - y\alpha_x(x) + \delta_z(y) + \delta_z(z),
 \end{aligned} \tag{9}$$

where P_x , P_y , and P_z are position errors in the x , y , and z directions, respectively.

Since

$$\begin{aligned}
 P_x &= E_x(x) + E_x(y) + E_x(z), \\
 P_y &= E_y(x) + E_y(y) + E_y(z), \\
 P_z &= E_z(x) + E_z(y) + E_z(z),
 \end{aligned} \tag{10}$$

we have

$$E_x(x) = \delta_x(x) - y^*\alpha_z(x) + z^*\alpha_y(x),$$

$$\begin{aligned}
 E_x(y) &= \delta_x(y) + z^*\alpha_y(y) - y^*\theta_{xy}, \\
 E_x(z) &= \delta_x(z) - z^*\theta_{xy}, \\
 E_y(x) &= \delta_y(x) - z^*\alpha_x(x), \\
 E_y(y) &= \delta_y(y) - z^*\alpha_x(y), \\
 E_y(z) &= \delta_y(z) - z^*\theta_{yz}, \\
 E_z(x) &= \delta_z(x) + y^*\alpha_x(x), \\
 E_z(y) &= \delta_z(y), \\
 E_z(z) &= \delta_z(z).
 \end{aligned} \tag{11}$$

Similar results can be obtained for other machine types.

G. Squareness errors

When the angles between xy , yz , and xz are not exactly 90° , then the diagonal distances can be expressed as

$$\begin{aligned}
 dR_{ppp} &= \theta_{xy}XY/R + \theta_{yz}YZ/R + \theta_{zx}ZX/R, \\
 dR_{npp} &= -\theta_{xy}XY/R + \theta_{yz}YZ/R - \theta_{zx}ZX/R, \\
 dR_{pnp} &= -\theta_{xy}XY/R - \theta_{yz}YZ/R + \theta_{zx}ZX/R, \\
 dR_{ppn} &= \theta_{xy}XY/R - \theta_{yz}YZ/R - \theta_{zx}ZX/R,
 \end{aligned} \tag{12}$$

where θ_{xy} is the squareness error in the xy plane, θ_{yz} is the squareness error in the yz plane, and θ_{zx} is the squareness error in the zx plane.

Solving Eq. (12), we have

$$\begin{aligned}
 \theta_{yz} &= (dR_{ppp} + dR_{npp})R/(2YZ), \\
 \theta_{zx} &= (dR_{ppp} + dR_{pnp})R/(2ZX), \\
 \theta_{xy} &= (dR_{ppp} + dR_{ppn})R/(2XY).
 \end{aligned} \tag{13}$$

By substituting the measured four diagonal displacement errors at the four end points into Eq. (13), the three squareness errors can be determined.

H. Error compensation

For most machine tools, the linear errors (sometimes called the pitch error or scale error) can be compensated for by the controller. However, there are other errors such as straightness errors (guide way straightness) squareness errors (squareness between axes), angular errors (pitch, yaw, and roll angles), and nonrigid body errors (weight shifting, counter balancing, etc.). Usually, the straightness errors and the squareness errors are much larger than the linear errors, hence only compensating for the linear errors is not enough.

Many controllers have the capacity to compensate for volumetric positioning errors, that is, to compensate for errors in the y direction and z direction as a function of x , compensate for errors in the x direction and z direction as a function of y , and compensate for errors in the x direction and y direction as a function of z . These correspond to the volumetric error components $E_y(x)$, $E_z(x)$, $E_x(y)$, $E_z(y)$, $E_x(z)$, and $E_y(z)$. Hence by inputting the measured volumetric error components to the controller the straightness

errors and the squareness errors can be compensated for and can reduce the machine tool positioning errors significantly.

Please note that the derivation of Eqs. (7) and (8) is very general and includes angular errors and nonrigid body errors. The volumetric errors determined here should be the same as the conventional straightness errors when the angular errors times the distance (between the volumetric error location and the conventional straightness location) is negligible compared to the repeatability of the machine [see Eq. (11)].

On the other hand, if better compensation is achieved by using the volumetric error components, this indicates that the method correctly solved the volumetric error components from the measured diagonal displacement errors. This is what we have found and reported in Part II of this article.

I. Sort data and order

The vector measurement technique collects data after each axis movement. Hence, three times more data are collected.

For each diagonal displacement data, sort out all position errors due to x -axis movement, y -axis movement, and z -axis movement. These measured position errors are $dR(x)_{ppp}$, $dR(y)_{ppp}$, and $dR(z)_{ppp}$ for the diagonal ppp , and similarly the same for other diagonals, i.e., npp , ppn , and ppn .

IV. MEASUREMENT ERRORS

In general, the measurement errors of a laser interferometer are small. Typical sources of error are alignment error or cosine error and errors of the air temperature sensor, the pressure sensor, and the material temperature sensor. The following are additional errors due to the laser vector measurement technique.

A. Projection error

The measured error is the linear errors projected to the body diagonal direction. For a cube, the length projected to the body diagonal is shortened by a factor of 1 over square root three or 0.577. Hence the minimum error or the least resolution is increased by a factor 1.732 larger than the laser interferometer measurement. For a rectangle, this projection factor may be larger, depending on the aspect ratio of the rectangle.

B. Laser beam alignment error

When the laser beam direction and the diagonal direction are not exactly parallel, there is a cosine error. If we assume the angle between the laser beam direction and the diagonal

direction is θ , the error is proportional to the square of θ and the distance traveled. For an alignment error of 1 mm over 1 m, the maximum error is less than 0.5 μm .

C. Flat-mirror alignment error

When the flat-mirror target is not exactly perpendicular to the laser beam, there is an error due to lateral motion of the flat mirror. If we assume the angle between the laser beam and the normal of the flat mirror is θ , the error is proportional to θ and the step size D . For an alignment error of 0.5 mm over 1 m, angle θ is equal to 0.5 mrad. If we assume a step size of 50 mm, the maximum error is less than 25 μm . This error is relatively large; however, because this error is a constant and nonaccumulative, it can be removed by data processing.

D. Error due to machine angular motion

Since the intersection of the laser beam and the flat mirror may not be the center of rotation of machine angular motion, the angular motion of the machine may generate large errors.

V. CONCLUSION

A new laser vector measurement technique for volumetric positioning errors was described. The experimental test in Part II of this article shows that, based on the diagonal displacement error measurements, there is a factor of 4-6 improvement with and without volumetric compensation. This is a good indication that the equations derived are correct and that the approximations used are valid.

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